

Entanglement Transfer Through an Antiferromagnetic Spin Chain

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We study the possibility of using a uniformly coupled finite antiferromagnetic spin-1/2 Heisenberg chain as a channel for transmitting entanglement. One member of a pair of maximally entangled spins is initially appended to one end of a chain in its ground state and the dynamical propagation of this entanglement to the other end is calculated. We show that compared to the analogous scheme with a ferromagnetic chain in its ground state, here the entanglement is transmitted faster, with less decay, with a much higher purity and as a narrow pulse form rising non-analytically from zero. Here non-zero temperatures and depolarizing environments are both found to be less destructive in comparison to the ferromagnetic case. The entanglement is found to propagate through the chain in a peculiar fashion whereby it hops to skip alternate sites.

Identifying potential methods for linking distinct quantum processors or registers is a crucial part of scalable quantum computing technology. Studying the potential of spin chains as quantum wires for the above purpose has recently emerged as an area of significant activity [1–5] as they can successfully transfer quantum states and Entanglement over short distance scales. One motivation for such wires is to circumvent the necessity of inter-conversion between solid state qubits and photons when connecting solid state quantum registers separated by short distances. Additionally, spin chains are systems of permanently coupled spins (essentially, a one dimensional magnet or isomorphic system). Thereby studying their potential to transfer quantum information automatically answers the question as to how well one can accomplish the transfer of a quantum state through a chain of coupled qubits without requiring the switchability or tunability of any of the interactions inside the chain – an example of *minimal control* in quantum information processing. This line of research can also be motivated simply as the study of canonical condensed matter systems from a quantum information perspective. As opposed to the hugely popular field examining how much entanglement exists inside such systems [6], this work investigates how quantum information *passes through* such systems.

In the original algorithm [1], as well as in most subsequent work [2–5], a chain of qubits (spin-1/2 systems) initialized in a fully polarized (symmetry broken) state plays the role of the channel. This would be the ground state, for example, if a ferromagnetic (FM) spin chain was used as the channel. The important noise factors such as the effects of temperature [4] and decoherence [5] have also been investigated for such FM channels. By now a plethora of physical implementations of such a scheme has either been performed using NMR [7] or suggested [8] (for Josephson junction arrays, trapped electron chains etc). However, how about using an antiferromagnetic (AFM) spin chain initialized in its ground state as a quantum channel for the transfer of entanglement? Strangely enough, the simplest version of this, namely a uniformly coupled spin-1/2 Heisenberg AFM chain as a channel for quantum information transfer, remains unstudied though examples of such chains are much more common than FM chains in condensed matter, including ones on which NMR studies are done [12]. They can be simulated in optical lat-

tices [13] and with Phosphorous doped Silicon [14]. Most strikingly, thanks to the progress of nanotechnology, antiferromagnetic (AFM) spin chains up to 10 spins in length have been built experimentally recently and the spin of the atoms and also the couplings between them can be probed individually by scanning tunneling microscopes [15]. This truly motivates an examination of the transfer of entanglement through AFM spin chains. Additionally, compared to FM channels, one can expect several qualitatively different features in AFM spin chain channels as they already have lot of entanglement inside, and the monogamous nature of shared entanglement may lead to nontrivial dynamics. Also, the channel is rotationally fully symmetric, and this leads to a qualitatively different channel for the transfer of quantum information.

Recently, the quality of state and entanglement transfer through all phases of a spin-1 chain (both FM and AFM) has been studied and some AFM phases have been shown to outperform the FM phases as a quantum wire [10]. Dimerized AFM states of Spin-1 chains can also enable certain state transfer schemes involving an adiabatic modulation of couplings [11]. It has also been shown that quantum information can be efficiently transferred between weakly coupled end spins of an AFM chain because of an effective direct coupling between these spins [9]. Some other recent studies of quantum state and entanglement transfer [17, 18] and entanglement dynamics [19] have considered initial states deviating from the usual fully polarized state. However, what about the simplest AFM chain of uniformly coupled spin-1/2 systems? In this letter, we obtain curious results about the propagation of entanglement through such a chain, in particular that it *hops to skip alternate sites* and that the entanglement transmitted through the channel rises from zero sharply and *non-analytically as a narrow pulse*. Such striking features will be very interesting to test with the finite AFM chains. In addition, we find that a channel with AFM initial state consistently outperforms the corresponding FM case for comparable chain lengths and reasonable times, even when temperature and decoherence effects are included.

We follow the approach of [1] to transfer entanglement from one end of an open AFM spin chain to the other and compare the quality and behavior in different situations with the case of FM. The Hamiltonian of the open chain with length

N_{ch} is

$$H_{ch} = J \sum_{i=1}^{N_{ch}-1} \sigma_i \cdot \sigma_{i+1}. \quad (1)$$

where the $\sigma_k = (\sigma_k^x, \sigma_k^y, \sigma_k^z)$ is a vector contains Pauli matrices which act on the site k and J is the coupling constant ($J > 0$ for AFM and $J < 0$ for FM). The protocol for transferring the entanglement is as follows: We place a pair of spins $0'$ and 0 in the singlet state $|\psi^-\rangle_{0'0} = \frac{1}{\sqrt{2}}(|0\rangle_{0'}|1\rangle_0 - |1\rangle_{0'}|0\rangle_0)$ while the channel (spins 1 to N_{ch}) is in its ground state $|\psi_g\rangle_{ch}$ (i.e. the ground state of H_{ch}). Note that for the AFM case, $|\psi_g\rangle_{ch}$ is a global singlet state of N_{ch} spins, while for the FM case it is a fully polarized ground state with all spins pointing in a given direction. Also note that both for the AFM chain for odd N_{ch} , and the FM chain, the ground state is non-unique and a unique ground state $|\psi_g\rangle_{ch}$ is selected out by applying an arbitrarily small magnetic field which does not affect the eigenvectors and just split up the degenerate energies. If one avoids applying the magnetic field to choose a unique ground state, any superposition of the degenerate eigenvectors could be chosen for the initial state. So that we can not get a unique result for comparison to other chains and also since in this situation the mixedness of the final state is increased the quality of entanglement goes down. When the initial state is prepared, we then turn on the interaction between spin 0 and first spin of the channel (spin 1). The Hamiltonian including this additional interaction is

$$H = I_{0'} \otimes (J\sigma_0 \cdot \sigma_1 + H_{ch}). \quad (2)$$

The total length of the system considered is thus $N = N_{ch} + 2$ with the total Hamiltonian being H (so that $0'$ never interacts with the channel) and the initial state being

$$|\psi(0)\rangle = |\psi^-\rangle_{0'0} \otimes |\psi_g\rangle_{ch}. \quad (3)$$

We are interested at the times that the entanglement between the spins $0'$ and N_{ch} peaks, which is the aim of the entanglement distribution through our spin chain channel. By turning on the interaction between spin 0 and the first spin of the channel (spin 1) the initial state evolves to the state $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$ and one can compute the density matrix $\rho_{0'N_{ch}} = \text{tr}_{0'N_{ch}}\{|\psi(t)\rangle\langle\psi(t)|\}$ where the meaning of tr_{ij} is the trace over whole of the system *except* sites i and j .

In figure 1 the entanglement (as quantified by the entanglement concurrence [16]) and the purity (as quantified by $\text{tr}(\rho_{0'N_{ch}}^2)$) of the state $\rho_{0'N_{ch}}$ for both the cases of AFM and FM chain as a function of time have been plotted for a system of length $N = 10$. It is clear from the figures that the behavior of the entanglement and the purity of the entangled state is completely different for the two cases. For the much studied FM case [1], the entanglement of the spins $0'$ and N_{ch} is simply equal to the modulus of the amplitude of an excitation to transfer from the site 0 to the site N_{ch} due to H which is always an analytic function. In contrast, in the AFM case we find a *nonanalytic* behavior in entanglement as a function of

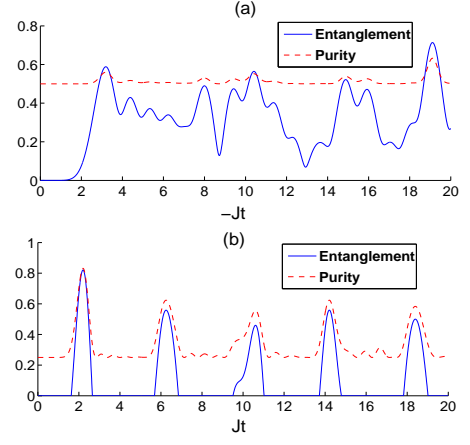


FIG. 1: (Color online) The concurrence and the purity of the state $\rho_{0'N_{ch}}$ for the chain of length 10. Figure (a) is for the case of FM chain ($J < 0$) and figure (b) is for AFM one ($J > 0$).

time. It is zero for most of the time and at regular intervals it suddenly grows up and makes a peak with its derivative being discontinuous at the point it starts to rise from zero. This behavior can be understood by realizing that the channel has to act as a purely depolarizing channel (equally probable random actions of all the three Pauli operators on a state while it passes through the channel) because of the $SU(2)$ symmetry of the channel state $|\psi_g\rangle_{ch}$. When one member of an entangled pair of qubits is transmitted through such a channel, then the two qubit state evolves to a Werner state [20]

$$\rho_{0'N_{ch}}(t) = p(t)|\psi^-\rangle\langle\psi^-| + (1 - p(t))I/4, \quad (4)$$

where I is the identity matrix for two qubits, and $p(t)$ is a time-dependent positive number ≤ 1 parameterizing the state. $\rho_{0'N_{ch}}$ will thus always be a Werner state with its parameter p varying with time. Initially p is zero (both qubits $0'$ and N_{ch} are maximally entangled to distinct systems $0'$ with 0 and N_{ch} with the rest of the chain respectively) and it rises from that as a simple trigonometric function of time. For example, for the simplest case $N_{ch} = 2$ (for which $|\psi_g\rangle$ is trivially a singlet and the starting state for the whole four qubit system is $|\psi(0)\rangle = |\psi^-\rangle \otimes |\psi^-\rangle$), one can analytically calculate the evolution easily to obtain $p(t) = \sin^2 2Jt$. It is known that as long as p remains $\leq 1/3$, the entanglement of the final state (4) stays constant at zero [20], and thereby the curve for entanglement versus time has a vanishing derivative. The entanglement starts to rise suddenly as soon as p exceeds $1/3$, but the trigonometric form of p (such as $\sin^2 2Jt$ for $N_{ch} = 2$) does not have a vanishing derivative (i.e., be in a maximum or minimum) at this point. There is thus a sudden discontinuity in the derivative of the curve of the entanglement of $0'$ and N_{ch} versus time. Though, finding a non-analyticity in the entanglement is not very interesting since the concurrence, similar to any other entanglement measure, has the source of non-analyticity in its definition but it is worthwhile to point out that entanglement gained by the FM chain is always ana-

lytic because in this case the correlation functions are bounded from below.

Another important difference between the case of FM and AFM chain that can be seen in figure 1 is the purity of the final entangled state. The purity of the state $\rho_{0'N_{ch}}$ is higher in the case of AFM chain in comparison with FM one. Having a purer entangled state transmitted is a distinct advantage as in the end one needs to purify the transmitted states by local actions to obtain a smaller number of states arbitrarily close to a singlet through a process called entanglement distillation [20]. Only such purified entanglement is really useful for linking distinct quantum processors and purer the shared entangled state, less is the effort to distill it. In addition, the very fact that $\rho_{0'N_{ch}}$ is a Werner state is a distinct advantage compared to the FM case. Werner states are a class of mixed states for which entanglement distillation methods are very well developed right from the start to the extent that in the original entanglement distillation paper [20] it was proposed to convert any mixed state to a Werner state first and then distill pure entanglement from it.

For the AFM chain with even number of spins since the final state is always a Werner state with the form of (4), entanglement and purity are uniquely determined by the parameter p . It is easy to show that the concurrence of the state (4) is $(3p - 1)/2$ and its purity is $(3p^2 + 1)/4$. For quantum state transferring and quantum communication one might prefer to directly send quantum states through the chain [1]. In this case one generates an arbitrary state $|\psi_s\rangle = \alpha|0\rangle + \beta|1\rangle$ at spin 0 while the chain (i.e., spins $1, 2, \dots, N_{ch}$) is in its ground state. Like the strategy explained above for entanglement distribution, at $t = 0$ the interaction between spin 0 and the rest is switched on. The dynamics of the system transfers the state $|\psi_s\rangle$ through the chain till it reaches the end. So then, at some proper times state of the last site $\rho_{N_{ch}}(t)$ is similar to $|\psi_s\rangle$. One can easily compute the fidelity $F = \langle\psi_s|\rho_{N_{ch}}(t)|\psi_s\rangle$ which is obviously a function of α, β and time t . One can average the fidelity F over all possible input states. This can be done by averaging over the surface of the Bloch sphere to get an input state independent parameter F_{av} . An straight forward computation gives $F_{av} = (3p + 1)/4$ for even AFM chains which is again determined uniquely by the Werner parameter $p(t)$ in Eq. (4). Thus these quantities, i.e., entanglement, purity and average fidelity, are not really independent and considering one of them provides enough information for the others so that we mainly focus on the entanglement in this paper.

To understand the difference between AFM and FM chains it is very important to notice that when the sign of the coupling J is changed the eigenvectors of the Hamiltonian do not change. So only the eigenvalues vary and consequently the ground state of the system changes. Our investigation shows that what is really important in the dynamics is the eigenvector which is chosen as the initial state and the sign of J is not important. It means that if for a AFM chain, which J is positive, we prepare a FM initial eigenvector, which all spins are aligned into a same direction, then the results are similar

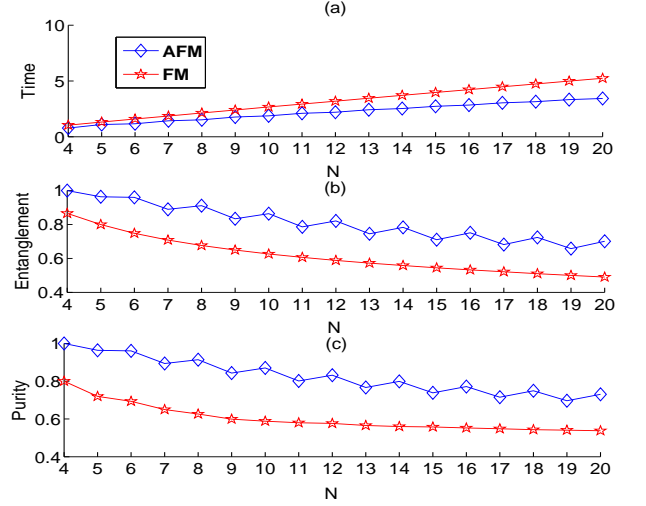


FIG. 2: (Color online) In this figure we have plotted the time of first maximum and the amount of entanglement and also purity versus length for both cases FM and AFM chains versus the length of the chain.

to a FM chain even though the Hamiltonian is AFM. Same results hold for the case that we generate the AFM eigenvector as the initial state of a FM chain. Using the AFM (FM) Hamiltonian for generating an AFM (FM) ground state have this benefit that simply with cooling the system it goes to its ground state while for an AFM (FM) chain generating a FM (AFM) eigenvector is practically very hard and needs lots of external control.

In practice, the time which one can afford to wait for the entanglement between $0'$ and N_{ch} to attain a peak is restricted by practical considerations such as the decoherence time of the system and simply by how much delay we can afford while connecting quantum processors. So we restrict ourselves to the case of the first maximum of the entanglement in time. In figure 2.a, we have plotted the time that entanglement achieves its first maximum value versus the total length of the system for chain lengths of up to $N = 20$ spins for both AFM and FM chains. It is clear that the speed of entanglement transmission through the AFM chain is higher than that through a FM chain independent of the length of the chain. In figure 2.b and 2.c, the amount of entanglement and purity in the first maximum of entanglement has been compared for both of the AFM and FM case, from which it is clear the the entanglement transmitted in the case of AFM chain has a higher value and also it's more pure than the entanglement transmitted in the case of FM chain. To see clearly the reason for the above superiority of the AFM chain over the FM chain, it is instructive to define something like a signal propagation wave in the two cases. This is because, in the end, it is the transfer of the state of spin 0 to spin N_{ch} that causes the entanglement to be set up between $0'$ and N_{ch} . In the case of the FM chain it is easy to define this as simply the propagation of a localized spin flip excitation (a superposition of all one magnon states) over a

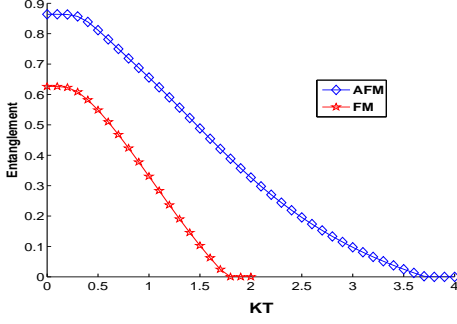


FIG. 3: (Color online) The amount of the first maximum of entanglement between two ends in a chain of length 10 versus the temperature for both the case of FM chain and AFM one.

polarized background state [1]. In the case of AFM chain it can be defined as a wave of modulation of the local density matrix of the spins if any state is appended to one end of the chain. For example, in an AFM ground state, the local density matrices of each spin will be the identity matrix. However, if a state $|1\rangle$ is appended to one end of it, and the system is allowed to evolve in time, there will be a wave of deviation of the local density matrices from the identity towards $|1\rangle\langle 1|$ which will propagate through the chain. This wave (for the AFM) simply travels faster through the chain than the spin flip excitation of a FM, and is responsible for the results of Fig. 2a. Additionally, this wave has a significantly lower dispersion than the corresponding case for the FM chain, which is responsible for the higher purity and higher entanglement for the AFM case.

Generally when the system is in non zero temperature, the state of the channel before evolution is described by a thermal state $\frac{e^{-\beta H_{ch}}}{Z}$ instead of the ground state, where $\beta = 1/KT$ and Z is the partition function of the channel. So in this case the initial state of the system is

$$\rho(0) = |\psi^-\rangle\langle\psi^-| \otimes \frac{e^{-\beta H_{ch}}}{Z} \quad (5)$$

and after time t the target state $\rho_{0'N_{ch}}(t)$ can be gained by

$$\rho_{0'N_{ch}}(t) = \text{tr}_{0'\hat{N}_{ch}} \{e^{-iHt} \rho(0) e^{iHt}\}. \quad (6)$$

In figure (3) we have plotted the value of the first maximum of concurrence of the state (6) for both the cases of FM and AFM chains in a system of length $N = 10$. The entanglement in the FM chains is more sensitive to the temperature and decays faster than AFM chain by increasing the temperature. The time at which the entanglement gets its first maximum is nearly independent of the temperature and changes slowly in agreement with [4].

In practical situations it is impossible to isolate the quantum systems from their environment. In the case of Markovian interaction between system and the environment a Lindblad equation describes the evolution of the system: $\dot{\rho} =$

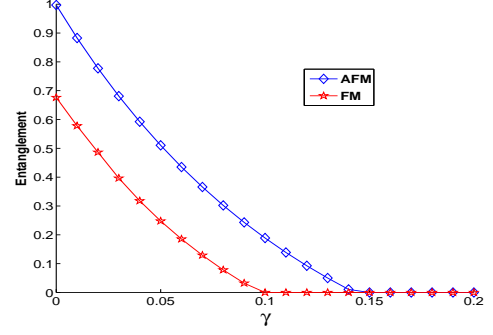


FIG. 4: (Color online) The amount of entanglement in its first maximum between two ends in a chain of length 6 versus the decoherence parameter γ in a fully polarized environment for both the case of FM chain and AFM one.

$-i[H, \rho] + \ell(\rho)$, where $\ell(\rho)$ is the Markovian evolution of the state ρ . In context of the situation we are studying it is reasonable to assume an environment which has no preferred direction. It is precisely for such an environment that a stable $SU(2)$ symmetric AFM ground state makes sense. Otherwise, for example, in an environment where some spin direction spontaneously decays to its opposite direction (an amplitude damping environment, in other words), an AFM state with approximately half the spins facing opposite to each other will decay into a symmetry broken FM ground state. Then the very premise of our investigation, namely starting from a AFM ground state loses meaning. Thus it is a reasonable assumption that the non unitary evolution $\ell(\rho)$ has the form

$$\ell(\rho) = -\frac{\gamma}{3} \sum_i \sum_{\alpha} \{\rho - \sigma_{\alpha i} \rho \sigma_{\alpha i}\}, \quad (7)$$

where index i takes $0', 0, \dots, N_{ch}$ and α gets x, y, z . The operators $\sigma_{\alpha i}$ means that the operator σ_{α} , which can be any of Pauli matrices, acting on the i th site of the whole system. The coefficient γ stands for the rate of decoherence in this dissipative environment. In figure 4, we have plotted the first maximum of entanglement versus γ for both the case of FM and AFM chains. In both cases, the entanglement decays exponentially with the decoherence parameter γ but the FM chain decays much more faster than AFM chain.

One can spot a simple but curious physical picture which describes the propagation of entanglement through the chains with even number of spins. Firstly, note that though one has the simplest possible spin-1/2 AFM chain (a uniformly coupled nearest neighbor chain) where one does not normally expect a dimer phase, the ground state is somewhat dimerized because of the “open ends” [21]. Thus if one takes an approach whereby one draws a bond for the presence of strong entanglement and no bond for very weak entanglement (< 0.1), the open ended AFM chain will be depicted as a dimerized state (though it is far from being an exact dimer). Appending a singlet of spins 0 and $0'$ at one end of the chain, makes the total system look like a series of strongly entangled

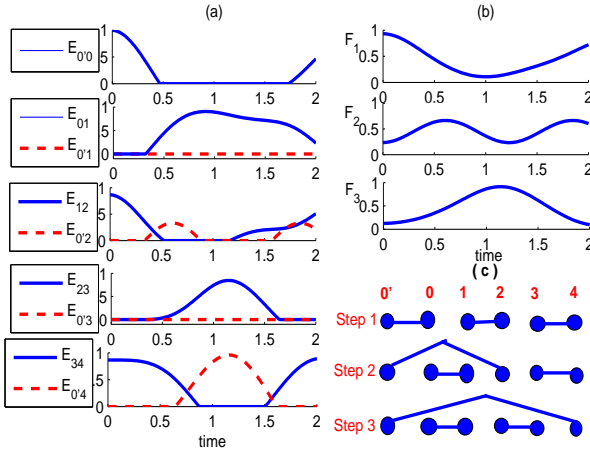


FIG. 5: (Color online) The entanglement between site $0'$ and the other sites $0, 1, 2, \dots, Nch$ during the time evolution in an AFM chain of length $N = 6$.

pairs next to each other and this is shown for the $N = 6$ case in step 1 of Fig. 5 c. The entanglements between $0'$ and any of the other spins of the chain, as well as the entanglement existing between the nearest neighbors for this chain are plotted in Fig. 5a. Surprisingly there is no entanglement at any time between site $0'$ and odd sites of the chain. The mode of propagation of entanglement through the spin chain is thus depicted in steps 1-3 of Fig. 5 c. Note that a bond drawn between site $0'$ and any of the other spins shown in the figure truly corresponds to the presence of entanglement between $0'$ and that spin (in other words, it is absent if there is no bond). Step 1 is an approximation of the initial state, while steps 2 and 3 are the times that spin $0'$ gets entangled with spins 2 and 4 respectively. The dynamics of entanglement of $0'$ hopping along the chain to skip alternate sites is generic for all even chains that we have considered. For the simplest case of $N = 4$, which can be analytically computed, the entanglement dynamics is simply an sinusoidal oscillation between the two states $|\psi^-\rangle_{0'0}|\psi^-\rangle_{12}$ and $|\psi^-\rangle_{0'2}|\psi^-\rangle_{01}$ with frequency $2Jt$ (a similar effect has been seen for spin-1 dimers and trimers in Ref.[10]). It is a generalization of this effect that we see for higher N . The curious dynamics depicted in figure 5c is, in fact, a very good approximation of the true dynamics even if the bonds were thought of as real singlets, and the overlap of that approximation with the “true” dynamics is shown in figure 5b.

In this paper we have examined the transfer of entanglement through AFM spin chains and found peculiar features including a nonanalytic behavior in the time variation of the transferred entanglement and a curious hopping mode of entanglement propagation skipping alternate sites of the channel. These predictions should be very interesting to test, potentially through local measurements on spins that can witness entanglement in an experiment (one such example re-

quires classically correlated measurements of spin operators in only three directions [22]), especially through NMR experiments [7, 12] or fabricated AFM nano-chains [15]. The amount of entanglement, purity and also its velocity of distribution in AFM is found to be superior to the case of FM chains, as well as the states being readily distillable. Furthermore AFM chains are more resistive to temperature and decoherence effects. It is an open question whether any of the plethora of techniques for perfecting the entanglement transfer in FM chains, such as coding and engineering [2, 3], have AFM analogs.

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